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Exciton–exciton scattering in semiconducting quantum well structures in the presence of a transverse electric field

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Abstract. Exciton–exciton elastic scattering in semiconducting quantum well structures in the presence of a transverse electric field is theoretically investigated. The scattering cross section is calculated for various electric field strengths and well widths. It is found that the electric field has little effect on the exciton–exciton scattering cross section in narrow quantum wells. On the other hand, in wide quantum wells the electric field significantly polarizes the excitons and enhances the total cross sections, which can exhibit a behaviour similar to that typical of free-carrier–free-carrier scattering in the quantum well. The effect of the electric field on the exciton–exciton scattering is found to be stronger than that on free-carrier–exciton scattering.

1. Introduction

Scattering of excitons by free carriers and by excitons as well as their contributions to broadening of the exciton linewidth in semiconductor quantum well structures have been investigated by various authors [1–6]. It is found that contributions to the exciton linewidth due to such scattering mechanisms can be significant in situations where high densities of free carriers and excitons are generated, and when the scattering of excitons by optical and acoustic phonons is reduced. Recently [7], we have investigated the elastic scattering of excitons by excitons in semiconducting quantum well structures using a finite-confining-potential model and found substantial enhancement of the scattering cross section in narrow quantum wells, with the emergence of a trend similar to that of bulk exciton scattering [8]. Such features are attributed to the quasi-3D nature of the exciton due to the substantial penetration of the exciton wavefunction into the barrier regions. In quantum wells of normal width, a trend similar to that predicted using a 2D model [9] is obtained.

In most device applications of the quantum well structures, an electric field is applied along the direction of confinement, which results in appreciable shifts in positions and broadening of excitonic absorption features in such structures [10–13]. It is thus of interest to study the effects of a transverse electric field on excitons in such structures, and more importantly, to investigate how these effects differ for quantum wells of different widths. Due to the fact that the width of the quantum well is narrow, the strength of the electric field is usually very large, even for a potential difference of a few volts applied across the quantum well. Therefore, perturbation theory is inappropriate in studying the effect of the strong electric field. In the past, various techniques [10–17], including the variational method [10–16], have been used to obtain approximate solutions for the exciton problem in quantum wells. It has been shown [16] that the two-parameter anisotropic relative wavefunction proposed by Matsuura and Kamizato [13, 18] produces the largest exciton binding energy for a quantum well of finite depth, even in the presence of the applied electric field. This trial wavefunction was used to obtain the exciton wavefunctions in our investigation of exciton–exciton elastic scattering in quantum wells [7].

Wu and Nurmikko [19] have performed calculations using the variational method and demonstrated the importance of including the effect of the electron–hole Coulomb interaction in determining the exciton lifetime against field ionization. They estimated that the exciton lifetime against field ionization is 20–100 times larger than what can be deduced from the one-particle calculations of Ahn and Chuang [17]. The electron–hole Coulombic interaction always tends to enhance the binding of the pair inside the well so as to reduce the tunnelling rate for a finite applied field. Hence for increasing electric field strengths, there could be a growing importance of effects due to exciton–exciton interactions in the quantum well relative to that due to free-carrier–exciton interactions, depending on the relative magnitudes of their cross sections in the presence of the field.

In this paper, we extend our calculation of the exciton–exciton elastic scattering cross section to include the effect of an electric field applied along the direction of carrier confinement. We again use the Born approximation to treat the scattering problem and the central-potential approximation for the interaction between the excitons. As is explained in [7], we will adopt the same semi-classical approach as Elkomos and Munschy [8, 20] used in their treatment of bulk exciton scattering, and consider only the symmetry effect when the two excitons involved are identical, while neglecting inter-exciton carrier exchange. We also note that in the previous work [7–9, 20], the exciton–exciton scattering cross sections could be expressed in terms of the electron-to-hole mass ratios of the excitons involved. Here, we shall consider the collisions of two types of exciton with electron-to-hole mass ratios of 0.15 and 0.8.

2. Calculation

We use a finite-confining-potential model similar to that used in the previous calculations for the well width dependence of exciton–exciton elastic cross sections [7]. However, in the presence of a uniform applied field, there are no longer truly bound states of the electron–hole system. Here we assume the quasi-static limit [21] in the presence of the electric field such that the quasi-bound states of the excitons remain within the well for a considerable amount of time to allow scattering to occur. Also, to account for the polarization effects in the transverse electric field, the variational wavefunctions describing the particles along the direction of confinement include a factor $\exp(\pm\beta_i z_i)$, such that the ground-state exciton wavefunction can be written as [15]

$$\psi(\boldsymbol{r}, z_e, z_h) = \frac{1}{\sqrt{N}}\phi_e(z_e)\phi_h(z_h)\phi_r(\boldsymbol{r}, z_e, z_h) \tag{1}$$

where

$$\phi_r(\boldsymbol{r}, z_e, z_h) = \exp\left[-\alpha\sqrt{r^2 + \gamma(z_e - z_h)^2}\right] \tag{2}$$

and

$$\phi_i(z_i) = \exp(\pm\beta_i z_i)\begin{cases} A_i \exp(-q_i|z_i|) & |z_i| > L/2 \\ \cos(k_i z_i) & |z_i| \leqslant L/2 \end{cases} \tag{3}$$

denote the 1s-like orbital for the relative in-plane radial motion between the electron and the hole, and the wavefunction for a charged particle confined in a one-dimensional quantum well in the presence of the electric field, respectively. In equation (3), the plus sign holds for the hole and the minus sign for the electron. It has been shown [13, 16] that this wavefunction closely approximates the behaviour of the exciton in the quantum well in the presence of the transverse electric field.

Using the quasi-2D formalism developed in [7] and the above wavefunctions, the differential cross sections for the elastic scattering of exciton $A(1, a)$ and exciton $B(2, b)$, consisting of hole–electron pairs $(a, 1)$ and $(b, 2)$ respectively, can again be written as

$$\sigma(\theta) = \frac{k}{k_0}\left|\sum_{i,j} f_{ij}(\theta)\right|^2 \tag{4}$$

for two excitons that are distinguishable and

$$\sigma(\theta) = \frac{k}{k_0}\left|\sum_{l,i} f_{li}(\theta) + f_{li}(\pi - \theta)\right|^2 \tag{5}$$

if the two excitons are identical. They are expressed as the summations of terms denoting the scattering amplitudes derived from the interactions between the particles of the incident and target excitons, as a result of their interacting potential, which in the central-field approximation can be written as

$$V_i = \frac{e^2}{\epsilon}\left(\frac{1}{|\boldsymbol{\rho}_{ab}|} + \frac{1}{|\boldsymbol{\rho}_{12}|} - \frac{1}{|\boldsymbol{\rho}_{a2}|} - \frac{1}{|\boldsymbol{\rho}_{1b}|}\right) \tag{6}$$

where $\boldsymbol{\rho}_{ij}$ is the 3D vector connecting the ith and jth particles. The scattering amplitude derived from the interaction between the ith particle of the incident exciton and the jth particle of the target exciton is given by

$$f_{ij}(\theta) = \pm\frac{\mu e^2 \exp(\mathrm{i}\pi/4)}{\epsilon\hbar^2\sqrt{2\pi k}}\int \mathrm{d}\boldsymbol{R}\ \exp[\mathrm{i}(\boldsymbol{k}_0 - \boldsymbol{k})\cdot\boldsymbol{R}]\left\langle\frac{1}{|\boldsymbol{\rho}_{ij}|}\right\rangle \tag{7}$$

where μ is the reduced mass of the combined two-exciton system and

$$\begin{aligned}\left\langle\frac{1}{|\boldsymbol{\rho}_{ij}|}\right\rangle = \frac{1}{N_A N_B}\int &\mathrm{d}\boldsymbol{r}_{a1}\ \mathrm{d}\boldsymbol{r}_{b2}\ \mathrm{d}z_1\ \mathrm{d}z_2\ \mathrm{d}z_a\ \mathrm{d}z_b\ \phi_1^2(z_1)\phi_2^2(z_2)\phi_a^2(z_a)\phi_b^2(z_b)\\ &\times \phi_{a1}^2(\boldsymbol{r}_{a1}, z_1, z_a)\phi_{b2}^2(\boldsymbol{r}_{b2}, z_2, z_b)\frac{1}{|\boldsymbol{\rho}_{ij}|}\end{aligned} \tag{8}$$

for $i \in \{a, 1\}$ and $j \in \{b, 2\}$. With the additional factors of $\exp(\pm\beta_i z_i)$ in the wavefunctions, all of the integrals above can still be solved analytically. However, the expressions are too long to be presented in full. The total elastic cross section is obtained by integrating over all scattering angles numerically:

$$\sigma = \int_{-\pi}^{\pi}\sigma(\theta)\ \mathrm{d}\theta. \tag{9}$$

3. Results and discussion

The total elastic cross sections for the scattering of identical excitons with an electron-to-hole mass ratio (m_e/m_h) of 0.15 is shown in figure 1 as a function of well width (L) and initial relative wavevector (k_0), for electric fields of 1 F_0 (a) and 5 F_0 (b), respectively. Here, the excitonic units are used for convenience. a_B is the exciton Bohr radius which is about 11 nm for the above electron-to-hole mass ratio and a relative dielectric constant

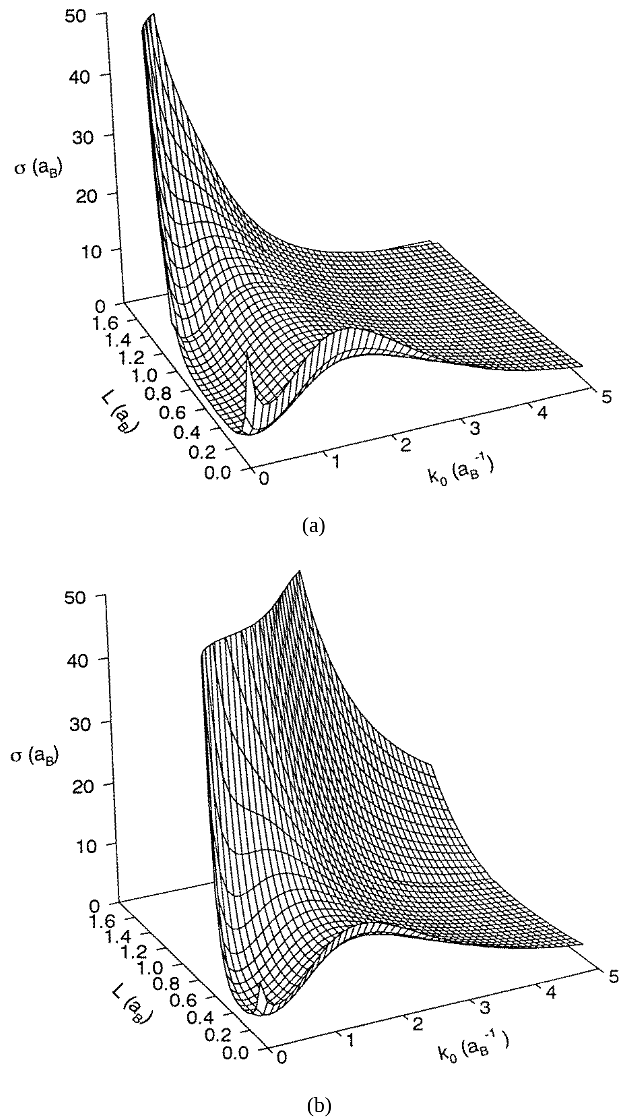
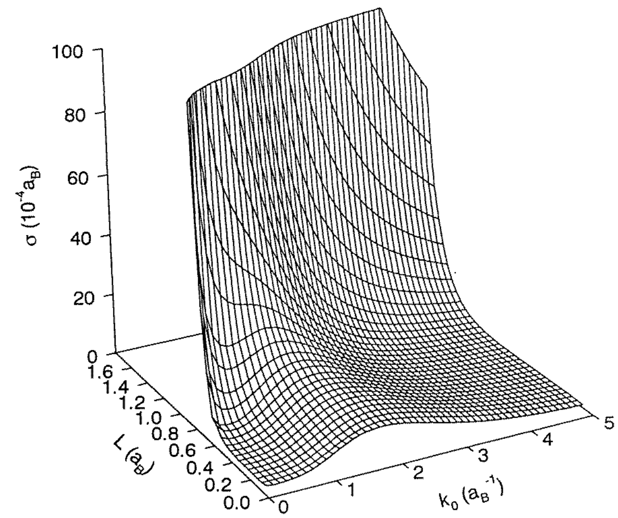


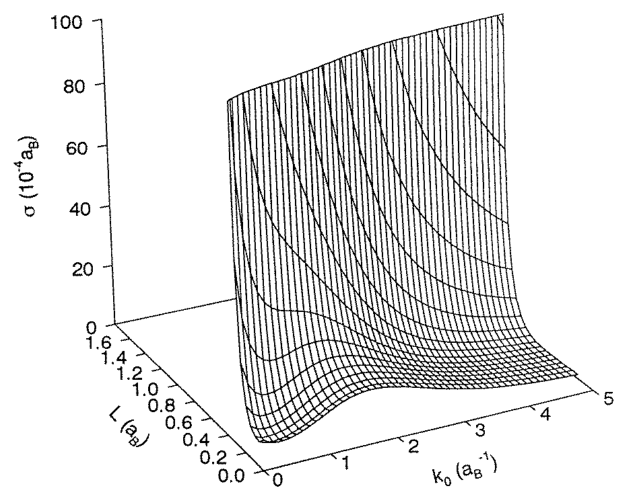
Figure 1. The total elastic cross sections with the symmetry effect, for the scattering of identical excitons with an electron-to-hole mass ratio of 0.15, are shown as functions of well width L and initial relative wavevector k_0 for electric fields of 1 F_0 (a) and 5 F_0 (b).

(ϵ) of 12.5, while the electric field is specified in terms of $F_0 = e/(\epsilon a_B^2)$ ($\sim$9 kV cm^{-1}). Figure 2 shows the total elastic cross section for the scattering of identical excitons with an electron-to-hole mass ratio of 0.8 for electric fields of (a) 1 F_0 and (b) 5 F_0. In this case, $a_B \sim 17$ nm and $F_0 \sim 4$ kV cm^{-1}. The corresponding total cross sections for elastic scattering between two different excitons having electron-to-hole mass ratios of 0.15 and 0.8 are shown in figure 3. Here, excitonic units for an electron-to-hole mass ratio of 0.15 have been used.

For all cases of exciton–exciton elastic scattering, by comparing parts (a) and (b) of the figures in each case, we see that the electric field has relatively little effect on the cross



(a)



(b)

Figure 2. The total elastic cross sections with the symmetry effect, for the scattering of identical excitons with an electron-to-hole mass ratio of 0.8, are shown as functions of well width L and initial relative wavevector k_0 for electric fields of 1 F_0 (a) and 5 F_0 (b).

section in narrow quantum wells ($L < 0.4a_B$). The exciton in such a narrow quantum well is hardly affected by the electric field, as the polarization of the exciton due to the electric field is negligible. This is also consistent with predictions made by Bastard *et al* [22] who found that the exciton wavefunction does not change much with the transverse electric field in the narrow wells. Therefore, even features typical of bulk exciton scattering in very narrow quantum wells remain almost the same in the presence of the electric field, and show only a slight increase with the field. This can be more clearly seen in the case of scattering of identical excitons of mass ratio 0.15 (by comparing figure 1 with the results in reference [7]), where these features are more obvious due to the electron-to-hole mass

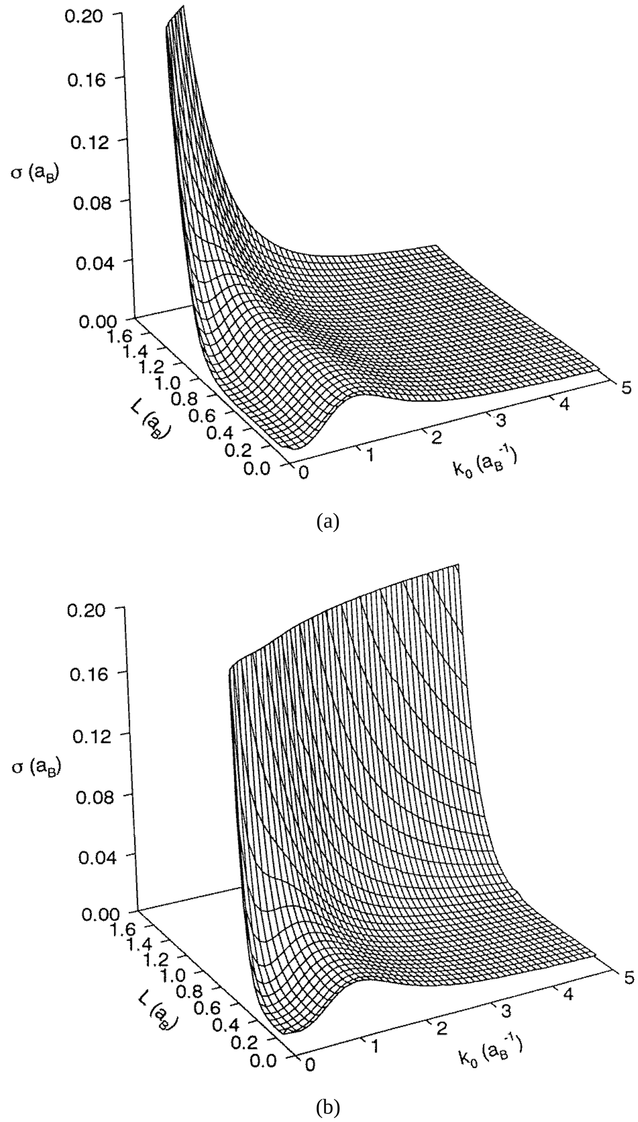


Figure 3. The total elastic cross sections, for the scattering of two different excitons with mass ratios of 0.15 and 0.8, are shown as functions of well width L and initial relative wavevector k_0 for electric fields of 1 F_0 (a) and 5 F_0 (b). Here, excitonic units of the exciton with a mass ratio of 0.15 are used.

ratio, as explained in [7].

The electric field, however, changes the magnitude as well as the behaviour of the total cross section as a function of k_0 in a relatively wide well. In the absence of the electric field, the total cross section increases as k_0 increases, before reaching a peak, beyond which the total cross section decreases for further increase in k_0. An increasing applied electric field enhances the overall total cross section dramatically. Furthermore, in the presence of the electric field, the total cross section gradually develops into a monotonically decreasing function of k_0, which is typical of free-carrier–free-carrier scattering [23]. These changes initially take place in the wider wells, and affect the narrower wells as electric field increases.

Physically, the reason for this free-carrier–free-carrier scattering behaviour is that the electric field polarizes the excitons significantly, so the scattering of the excitons is dominated by interactions between like particles. This phenomenon can also be interpreted from the expression of the scattering amplitude

$$f(\theta) = f_{ab} + f_{12} - f_{a2} - f_{1b} \tag{10}$$

where each of the terms can be further factored into portions describing in-plane contributions which we denote as $\chi_{ij}^{\parallel}$, and contributions due to the interaction along the direction of carrier confinement, $\chi_{ij}^{\perp}$, i.e.

$$f_{ij}(\theta) = \pm \exp\left(\mathrm{i}\frac{\pi}{4}\right)\frac{(2\pi)^{5/2}\mu e^2}{\epsilon\hbar^2}\frac{\chi_{ij}^{\parallel}\chi_{ij}^{\perp}}{\Delta k\sqrt{k}N_{a1}N_{b2}} \tag{11}$$

where $\Delta\boldsymbol{k}$ is the change in the relative wavevector of the excitons due to the scattering. $\chi_{ij}^{\parallel}$ and $\chi_{ij}^{\perp}$ are obtained by integrating over the in-plane coordinates $\boldsymbol{r}$ and the z-coordinates respectively. For example, when we have scattering of the holes associated with the two excitons by each other,

$$\chi_{ab}^{\parallel} = \frac{4\alpha_1\alpha_2}{\left\{\left[4\alpha_1^2 + (m_1\,\Delta k/m_A)^2\right]\left[4\alpha_2^2 + (m_2\,\Delta k/m_B)^2\right]\right\}^{3/2}} \tag{12}$$

$$\chi_{ab}^{\perp} = \langle\phi_a\phi_b|Q_1(z_a)Q_2(z_b)\exp(-\Delta k|z_b - z_a|)|\phi_a\phi_b\rangle \tag{13}$$

and

$$Q_1(z_a) = \langle\phi_1|\left\{1 + \sqrt{\gamma_1\left[4\alpha_1^2 + (m_1\,\Delta k/m_A)^2\right]}\,|z_a - z_1|\right\} \times \exp\left\{-\sqrt{\gamma_1\left[4\alpha_1^2 + (m_1\,\Delta k/m_A)^2\right]}\,|z_a - z_1|\right\}|\phi_1\rangle \tag{14}$$

$$Q_2(z_b) = \langle\phi_2|\left\{1 + \sqrt{\gamma_2\left[4\alpha_2^2 + (m_2\,\Delta k/m_B)^2\right]}\,|z_b - z_2|\right\} \times \exp\left\{-\sqrt{\gamma_2\left[4\alpha_2^2 + (m_2\,\Delta k/m_B)^2\right]}\,|z_b - z_2|\right\}|\phi_2\rangle \tag{15}$$

where m_A and m_B are the effective masses of the two excitons, and m_1 and m_2 are the effective masses of the electrons. The expressions for the other terms are similar, with the only differences being the masses and the argument of the exponential factor in equation (13). Even though the exciton–exciton scattering amplitude in the central-field and Born approximations can always be written in the above form, with or without the electric field, the relative strength of these terms as well as those of $\chi_{ij}^{\parallel}$ and $\chi_{ij}^{\perp}$ are strongly affected by the electric field in a relatively wide quantum well. One major factor through which the electric field influences the scattering cross section is through the $\exp(-\Delta k|z_i - z_j|)$ term in equation (13), which increases when the ith and the jth particles are of the same type (both electrons or both holes) such that they are pushed to the same side of the quantum well by the electric field, hence enhancing their interaction along the z-axis. Correspondingly, if the ith and the jth particles are of opposite charges, then the factor $\exp(-\Delta k|z_i - z_j|)$ would bring about a decrease of $\chi_{ij}^{\perp}$, and hence a reduction in the interaction of unlike particles. It should be noted that the extent of these effects depends on both the strength of the field and the width of the well.

In the absence of the electric field, and for well width $>0.5a_B$, the typical 2D behaviour of the exciton–exciton total elastic cross section, which increases as k_0 increases until it reaches a peak beyond which it decreases with further increase in k_0, can be largely attributed to the in-plane contribution $\chi_{ij}^{\parallel}$, as was discussed in [7]. However, in the presence of the electric field which polarizes the exciton in a wide quantum well along the direction of the carrier confinement, the $\chi_{ij}^{\perp}$-terms gradually gain importance. This interchange of the dominance of the $\chi_{ij}^{\parallel}$- and $\chi_{ij}^{\perp}$-factors within the scattering amplitude brings about the changes in behaviour of the cross sections for different well widths and electric field strengths. As the electric field increases in a wide well, the increasing differences between the $\chi_{ij}^{\perp}$-factors are able to offset the decreasing trend caused by $\chi_{ij}^{\parallel}$ in the region of small incident energies where the effects of the $\chi_{ij}^{\parallel}$-factors are weaker, to give an increase in cross sections, which further results in the monotonically decreasing trend of the total cross sections in wide wells and strong electric fields. Hence, in the presence of an increasing electric field which polarizes the excitons and enhances (reduces) the interaction between like (unlike) particles, the exciton–exciton elastic cross section in a wide well approaches that of free-carrier–free-carrier scattering, before the excitons are field ionized.

Previously, in [7], we showed that the magnitude of the exciton–exciton elastic scattering cross section in the absence of the electric field is very sensitive to the electron-to-hole mass ratio. Here, comparisons of figures 1, 2 and 3 for the different scattering processes show that the exciton–exciton elastic cross section for a mass ratio of 0.8 increases much more quickly with the electric field than those for a mass ratio of 0.15; and the monotonically decreasing trend develops at a smaller well width and electric field strength. This might be due to the fact that an exciton with a larger electron-to-mass ratio (i.e. smaller hole mass) is more easily polarized by the field since its binding energy is smaller than that of one with a smaller mass ratio. The scattering cross section of two different excitons having mass ratios of 0.15 and 0.8 shows a magnitude and behaviour intermediate between those of the identical-exciton scatterings. However, we note that excitons with a larger mass ratio might be field ionized in a weaker electric field than would be required to ionize those with a smaller mass ratio, though in the calculations we have neglected this possibility. Miller *et al* [10] have found that the excitonic features in quantum wells remain resolvable in electric fields up to $\sim$100 kV cm^{-1}.

It has been found [23] that for free-carrier–exciton scattering, the hole–exciton elastic cross section also increases with the electric field, and is larger than that for electron–exciton scattering. In the absence of the electric field [7], the exciton–exciton elastic cross section is smaller than that for hole–exciton scattering, though it is larger than that of electron–exciton scattering, for a fixed well width and a mass ratio of 0.15. However, as the electric field now maps both particles of the incident exciton onto particles of similar charges in the target exciton, the effect of the electric field on exciton–exciton scattering is nearly twice that of the free-carrier–exciton scattering [23]. In the presence of the electric field, there is an enhancement in the overall interactions between the particles of similar charges as denoted by the increase in f_{ab} and f_{12} in the scattering amplitude, as well as a reduction in the cancellation effect due to interaction between particles of opposite charges, denoted by f_{a2} and f_{1b}, such that the cross section increases more dramatically as compared with the case of a single incident particle for free-carrier–exciton scattering. There, the scattering amplitude consists of only two terms, f_e and f_h, due to interaction of the incident free carrier with the electron and hole of the exciton respectively [23]. This results in a more rapid increase in cross section with well width and electric field for exciton–exciton scattering, which can again be seen by comparing the present exciton–exciton elastic cross sections with those for the hole–exciton scattering [23] for a mass ratio of 0.15 and the same electric field strength. The monotonically decreasing features typical of free-carrier–free-carrier scattering also appear at smaller well widths and electric field strengths for exciton–exciton scattering than for free-carrier–exciton scattering.

From the faster rate of increase in magnitude of the cross section with electric field for exciton–exciton scattering as compared with that due to hole–exciton scattering, the initially smaller cross sections for the exciton–exciton scattering can increase up to the same order of magnitude as that for the hole–exciton scattering or even exceed it, under appropriate conditions. Therefore it is possible for the effects due to exciton–exciton interactions to become comparable to if not surpass that due to the free-carrier–exciton interactions in the presence of the electric field.

4. Conclusion

In conclusion, we have studied the cross sections due to elastic scattering of excitons by excitons in semiconducting quantum wells in the presence of a transverse electric field. It is found that the electric field significantly enhances and alters the behaviour of the scattering cross section in wide quantum wells, particularly in the low-energy region, while the effect in narrow quantum wells is small. This enhancement in the cross sections is expected to further broaden the exciton linewidth in such structures.

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